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Wavelets, Signal Processing and Matrix Computations

FINAL REPORT

Final report for the period 1 October 1993 to 15 September 1994

Bruce W. Suter, Ph.D.

Department of Electrical and Computer Engineering (AFIT/ENG

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1 Key Scientific Results

From 1 October 1993 to 15 September 1994 .. Suter and his research group have achieved research results in the following four important areas: Multidimensional Malvar Wavelets; Time/Spatial Varying Filter Banks; Vector Filter Banks and Vector-Valued Wavelets; and Computational Multirate and Multirate Time-Frequency Analysis.

References in this section will be identified as either background references, which are with identified by a number prefixed by an "R", or references to new results, which are identified as numbers without a prefix. Citations for backgro. .d references are given at the end of this section; while, citations for new results are gi. .. . ction 3 on Publications and Presentations.

A. Multidimensional Malvar Wavelets

Malvar[R1][R2] recently developed the Lapped Orthogonal Transforms (LOT), which was introduced by Cassereau[R3] to eliminate thr blocking effects in transform coding. LOTs are called Malvar wavelets[R4] in wavelet literature. With Malvar wavelets, a given signal can be decomposed into a linear combination of time-frequency atoms and moreover the signal can be perfectly reconstructed from the coefficients in the decomposition. A general description of window functions for Malvar wavelets was given by Coifman and Meyer[R5]. With this description of window functions Malvar wavelets were generalized by Suter and Oxley[R6] and Auscher, Weiss and Wickerhauser[R7] to more general local orthonormal bases, such as orthonormal polynomials, wavelets on intervals, etc. Since one dimensional Malvar wavelets are now well understood, the next challenge is to understand multidimensional Malvar wavelets. Moreover multidimensional Malvar wavelets are not only interesting in theory but also in applications, such as image processing, simulations of turbulence flow and etc. A trivial way to construct multidimensional Malvar wavelets is the tensor product of (separable) one dimensional Malvar wavelets. However, in many applications nonseparable Malvar wavelets are more important than separable Malvar wavelets. For instance, using separable Malvar wavelets to represent turbulent flows has been shown to be unsatisfactory, because the corners of the rectangular regions act as additional sources of turbulence[R8]. Thus nonseparable Malvar wavelets are preferred solution for this problem, but construction of nonseparable Malvar wavelets has been considered, up to this point, as an open problem. In [1], we opened a new area of research by constructing a family of two dimensional nonseparable Malvar wavelets. This work opens a new area of research on multidimensional Malvar wavelets. In addition, in [10] we constructed a family of Malvar wavelets on hexagons, which could be very important in many areas, such as computer graphics, numerical analysis, image processing and etc.

B. Time/Spatial Varying Filter Banks

Multirate filter bank theory has been extensively studied in recent literature due to its wide applications in signal analysis, processing and coding[R9]. As such, two special cases of filter banks, Discrete Wavelet Transforms and Discrete Fourier (or Cosine, Sine) Transform have been found to be particularly important in speech[R10] and image compression[R11]. A characteristic of conventional filter banks is that the filters do not change with time. Because of the nonstationarity of signals, such as, speech, audio and images, time/spatial varying filter banks have been desired by recent researchers to exploit the nonstationarity, where filter banks are allowed to be changed dynamically with the nature of a signal. An important problem in time-varying filter bank theory is how to construct time-varying filter banks with the perfect reconstruction (PR) property from local PR filter banks. For this problem, there have been several papers and presentations[R12][R13][R14]. However, their approaches are ad hoc. And, only a few of them deal with spatial-varying filter banks due to their associated complexity.

In [2], we presented a systematic theory to construct perfect reconstruction (PR) time-varying FIR multirate filter banks with overlaps by using arbitrary FIR multirate PR filter banks in different time intervals. We constructed proper window functions cooperating with the even and odd extensions of filters at the boundaries of the intervals. We presented conditions on such window functions for the perfect reconstruction property of the global filter banks in the construction, which are independent of local filter banks. From this theory, one is able to use discrete wavelet transforms, discrete cosine transform and several other transforms on different time intervals. The theory can be applied to both infinite and finite length signals and moreover the blocking effects in the transitions between neighboring filter banks have been eliminated. A numerical example using a discrete cosine transform and a discrete wavelet transform on different time intervals of different length was presented to illustrate the theory.

In [6], we presented a systematic method to construct two dimensional spatial-varying filter banks with perfect reconstruction property. These filter banks permit two dimensional separable or nonseparable local bases on different rectangular regions, where overlaps between neighboring regions are used to eliminate the blocking effects. In particular, nonseparable discrete wavelet transforms were studied. A numerical example was also given to illustrate the theory.

C. Vector Filter Banks and Vector-Valued Wavelets

The conventional multirate filter bank theory, wavelet theory and orthogonal transforms are well understood. However, possible improvements were investigated from several aspects as follows.

First of all, it is known that there is a limitation for the time-frequency localization of a single mother wavelet, that is, if it is localized in the time domain then it will not be localized in the frequency domain. Recently, Geronimo, Hardin and Massopust[R15] constructed two functions $\psi_1(t)$ and $\psi_2(t)$ whose translations and dilations form an orthonormal basis for $L^2(\mathbf{R})$. The importance of these two functions is that they are continuous, symmetric, and time-localized (or short support). Moreover, Geronimo, Hardin, and Massopust's research tells us that, if we allow several mother wavelets (or multiwavelets) in an expansion, then we may have better properties than those for a single wavelet function.

Second, vector transforms have been recently introduced for image coding by Weiping Li[R16], where input and output signals are finite vectors with same dimension. It was shown that vector transforms are advantageous in image coding at low bit rates. Moreover, vector quantization can be applied more easily in the vector transform domain than in the conventional scalar transform domain. This is because the local correlation between samples is exploited more optimally when blocked signals are used. This is similar to the reason why the performance of vector quantization is better than the performance of scalar quantization in general.

Thirdly, multirate filter banks with block sampling were recently studied by Khansari and Leon-Garcia[R17], where the traditional uniform down/up sampling in multirate filter banks is replaced by block down/up sampling. They showed that an FIR analysis filter bank may not have an FIR synthesis filter bank with traditional down/up sampling so that the system satisfies the perfect reconstruction (PR) property; However, it may have an FIR synthesis filter bank with block down/up sampling so that the system has PR property. Unfortunately, their approach is ad hoc and a limited number of results were obtained.

In [8], we studied general vector filter banks where the input signals and transfer functions in conventional multirate filter banks are replaced by vector signals and transfer matrices, respectively. We showed that multirate filter banks with block sampling and linear time invariant transfer functions studied by Khansari and Leon-Garcia are special vector filter banks where the transfer matrices are pseudo-circulant. We presented some fundamental properties for the basic building blocks, such as, Noble identities, interchangeability of down/up sampling, polyphase representations of M-channel vector filter banks and multirate filter banks with block sampling. We then presented necessary and sufficient conditions for the alias free property, FIR systems with FIR inverses, paraunitariness and lattice structures for paraunitary vector filter banks. We also presented a necessary and sufficient condition for paraunitary multirate filter banks with block sampling. As an application of the theory, we presented all possible perfect reconstruction delay chain systems with block sampling. We showed some examples which are not paraunitary for conventional multirate filter banks but are paraunitary for multirate filter banks with proper block sampling. In this paper, we also presented a connection between vector filter banks and vector transforms, which has applications in image coding. This work opens a new area in multirate vector filter banks and vector transforms.

In [9], we introduced vector-valued multiresolution analysis and vector-valued wavelets for vector-valued signal spaces. We constructed vector-valued wavelets by using paraunitary vector filter bank theory developed in [8] and [3]. In particular, we constructed vector-valued Meyer wavelets that are band-limited. We classified and constructed vector-valued wavelets with sampling property. As an application of vector-valued wavelets, multiwavelets can be constructed from vector-valued wavelets. We showed that certain linear combinations of known scalar-valued wavelets may yield multiwavelets. We then presented discrete vector wavelet transforms for discrete-time vector-valued (or blocked) signals, which can be thought of as a family of unitary vector transforms. In applications of vector wavelet transforms in two dimensional transform theory, the nonseparability can be easily handled. This work opens a new area for vector-valued wavelets, constructions and fast implementations of multiwavelets and two dimensional transform theory.

D. Computational Multirate and Multirate Time-Frequency Analysis

Multirate systems, which find application in the design and analysis of filter banks, are demonstrated to be useful as a computational paradigm [14]. It is shown that any problem which can be expressed as a set of vector-vector, matrix-vector or matrix-matrix operations can be recast using multirate. This means that numerical linear algebra can be recast using multirate as the underlying computational paradigm. By viewing multirate as a computational paradigm, many problems found in signal processing can also be reformulated into fast parallel algorithms. For example, this paradigm is applied in a straight forward fashion to the Fast Fourier Transform (FFT) and the Discrete Hartly Transform (DHT) to create fast parallel versions of these algorithms. This work opens a new area of computational multirate.

As a non-trivial example, the multirate computational paradigm is applied to the problem of Generalized Discrete Time-Frequency Distributions (GDTFD) to create a new family of fast algorithms for the calculation of Time-Frequency Distributions (TFD). The application of multirate as a computational paradigm to GDTFD's forms a new class of distributions called the Decimated GDTFD (D-GDTFD) [13]. These distributions, which are based upon the Zak transform[R18][R19], trade bandwidth for speed. For a decimation factor of m, there is an m fold increase in throughput (or speed of calculation). The corresponding reduction in discrete bandwidth is from 2π for the GDTFD to $2\pi/m$ for the D-GDTFD. An important attribute of the D-GDTFD is that it requires significantly less storage than the GDTFD. The D-GDTFD requires only $1/m^2$ of the storage of the GDTFD. By combining several D-GDTFD's, it is possible to reconstruct a GDTFD. This reconstruction of D-GDTFD's is the Multirate Time-Frequency Distribution (MRTFD) [15]. Each D-GDTFD is independent, and as a result, the MRTFD can easily be implemented in parallel for an increase in throughput on the order of m. If additional parallel paths are available, the individual D-GDTFD's can also be implemented in parallel leading to improvements in throughput on the order of m^2 or more. Two distinct MRTFD algorithms were developed [15]. The first MRTFD is based upon the inner product form of the GDTFD and combines the Zak transform, weighted spectrograms[R20] and Singular Value Decomposition (SVD)[R21]. It is called the SVD MRTFD and calculates the distribution for particular instants of time. The second MRTFD is based upon the outer product form of the GDTFD and is called the Circular Convolution MRTFD. It also builds upon the Zak transform and calculates the distribution for blocks of time instead of isolated instants. This work opens a new area of multirate time-frequency analysis.

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2 Technology Transfer

Many Air Force and commercial applications will be benefit from these results. One Air Force application is in the area of reconnaissance–namely, the ability to efficiently transmit video images while a mission is in progress. This same technology could also be applied to enhance High Definition Television (HDTV) and, possibly, products in the emerging area of multimedia.

Presently, WL/ELED is developing a prototype chip for scalar-valued Malvar wavelets. WL/ELED has been talking with the F22 SPO concerning the application of scalar-valued Malvar wavelet chip in a data compression application that involves radar modeling.

The exercise of transferring basic research to other Air Force organization illustrated that to efficiently disseminate multirate and wavelet signal processing results, Air Force officers need to be better educated in signal processing. Early in 1994, approval was granted for a new education code in this area of signal processing, designed 4IPY. This program will permit masters and doctoral students to come to AFIT for the purpose of studying signal processing. With future graduates in the laboratories, the SPOs, and throughout the Air Force, the dissemination of basic research results will be much easier.

3 Publications and Presentations

A. Journal Papers (Accepted and Submitted)

[1] X.-G. Xia and B. W. Suter, A family of two dimensional nonseparable Malvar wavelets, *Applied and Computational Harmonic Analysis*, submitted.

[2] X.-G. Xia and B. W. Suter, A systematic method for the construction of time-varying FIR multirate filter banks with perfect reconstruction, *IEEE Transactions on Signal Processing*, submitted.

[3] X.-G. Xia and B. W. Suter, FIR paraunitary filter banks given several analysis filters: factorizations and constructions, *IEEE Transactions on Signal Processing*, submitted.

[4] X.-G. Xia, B. W. Suter, and M. E. Oxley, On necessary and sufficient conditions for perfect reconstruction multidimensional delay chain systems, *IEEE Transactions on Signal Processing*, submitted.

[5] X.-G. Xia and B. W. Suter, On the Householder transform in $\mathbf{C}^m$, *Digital Signal Processing*, to appear.

[6] X.-G. Xia and B. W. Suter, On the construction of two dimensional perfect reconstruction FIR spatial-varying filter banks with overlaps, *IEEE Transactions on Signal Processing*, submitted.

[7] X.-G. Xia, B. W. Suter and M. E. Oxley, Malvar wavelets with asymmetrically overlapped windows, *IEEE Transactions on Signal Processing*, submitted.

[8] X.-G. Xia and B. W. Suter, Multirate filter banks with block sampling, *IEEE Transactions on Signal Processing*, submitted.

[9] X.-G. Xia and B. W. Suter, Vector-valued wavelets and vector filter banks, *IEEE Transactions on Signal Processing*, submitted.

[10] X.-G. Xia and B. W. Suter, On constructions of Malvar wavelets on hexagons, *Applied and Computational Harmonic Analysis*, submitted.

[11] X.-G. Xia, Comments on "An Optimum Complete Orthonormal Basis for Signal Analysis and Design", *IEEE Transactions on Information Theory*, submitted.

[12] J. R. O'Hair and B. W. Suter, Kernel Design Techniques for Alias-Free Time-Frequency Distributions, *IEEE Transactions on Signal Processing*, submitted.

[13] J. R. O'Hair and B. W. Suter, The Zak Transform and Decimated Time-Frequency Distributions, *IEEE Transactions on Signal Processing*, submitted.

[14] J. R. O'Hair and B. W. Suter, Multirate: A New Computational Paradigm, *IEEE Transactions on Signal Processing*, submitted.

[15] J. R. O'Hair and B. W. Suter, Multirate Time-Frequency Distributions, *IEEE Transactions on Signal Processing*, submitted.

B. Refereed Book Chapters

[16] B. W. Suter and M. E. Oxley, Getting Around the Balian-Low Theorem Using Generalized Malvar Wavelets, in L. Puccio, L. Montefusco, and C. Chui(editors), *Wavelets: Theory, Algorithms, and Applications*, to appear.

[17] X.-G. Xia and B. W. Suter, On the construction of two dimensional spatial-varying FIR filter banks with perfect reconstruction, in S. P. Singh(editor), *Recent Developments in Approximation Theory: Wavelets and Applications*, to appear.

C. Conference Papers and Presentations

[18] B. W. Suter and M. E. Oxley, Getting Around the Balian-Low Theorem, *International Conference on Wavelets*, Taormina, Italy, October 1993.

[19] X.-G. Xia, C.-C. Jay Kuo and B. W. Suter, Improved Backus-Gilbert method for signal reconstruction with a wavelet model, *SPIE Proceedings*, Vol. 2242, pp. 420–431, Orlando, Florida, April 1994.

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[24] X.-G. Xia and B. W. Suter, On Constructions of Two Dimensional Nonseparable Malvar Wavelets, *Proceedings of SPIE, Mathematical Imaging: Wavelet Applications in Signal and Image Processing*, San Diego, California, July 1994.

D. Invited Presentation

[25] B. W. Suter and M. E. Oxley, The Balian-Low theorem via complex-valued Malvar wavelets, *Symposium on Applications of Subbands and Wavelets*, Newark, NJ, March 1994.

E. Conference Papers (Accepted and Submitted) for Next Fiscal Year

[26] X.-G. Xia and B. W. Suter, Construction of perfect reconstruction time-varying FIR multirate filter banks with overlaps, *Proceedings of IEEE-SP International Symposium on Time-Frequency and Time-Scale Analysis*, Philadelphia, October 1994, to appear.

[27] J. R. O'Hair and B. W. Suter, Decimated time-frequency distributions, *Proceedings of IEEE-SP International Symposium on Time-Frequency and Time-Scale Analysis*, Philadelphia, October 1994, to appear.

[28] X.-G. Xia and B. W. Suter, Vector filter banks and multirate filter banks with block sampling, *Proceedings of the 28th Annual Asilomar Conference on Signals, Systems, and Computers*, Pacific Grove, California, October 1994, to appear.

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[30] X.-G. Xia and B. W. Suter, Vector-valued multiresolution analysis and vector filter banks, *IEEE International Conference on Acoustics, Speech and Signal Processing*, Detroit, Michigan, May 1995, submitted.

[31] J. R. O'Hair and B. W. Suter, Multirate time-frequency distributions, *IEEE International Conference on Acoustics, Speech and Signal Processing*, Detroit, Michigan, May 1995, submitted.

[32] X.-G. Xia and B. W. Suter, Malvar wavelets on Hexagons, *Eighth International Conference On Approximation Theory*, College Station, Texas, January 1995, submitted.

4 Signal Processing Seminar Series

AFOSR/NM sponsored a Signal Processing Seminar Series at the Air Force Institute of Technology during this past year. The talks given as part of that series are listed below.

Leon Cohen (Hunter College and Graduate Center of CUNY) "What is Scale," presented on 18 November 1993.

Gregory Beylkin (University of Colorado at Boulder) "Implementation of Operators via Multirate Filter Banks," presented on 20 January 1994.

Henrique Malvar (PictureTel) "Recent Results in the Theory and Application of Malvar Wavelets," presented on 3 March 1994.

Jeffrey Geronimo (Georgia Tech) "Intertwining Multiresolution Analysis and the Construction of Multiwavelets," to be presented on 29 September 1994.

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Key scientific results in the following four areas: (1) multidimensional Malvar wavelets; (2) time/spatial varying filter banks; (3) vector filter banks and vector-valued wavelets; and (4) multirate time-frequency. These results have opened the following new areas of research: nonseparable multidimensional Malvar wavelets, vector-valued wavelets and vector filter banks, and multirate time-frequency analysis. These results also provide fundamental tools in many Air Force and industrial applications, such as modeling of turbulence, compression of images/video images, etc.

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